

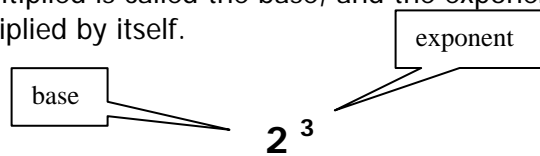
**ENGINEERING GATORTRAX MATH EXCELLENCE PROJECT**  
**ENGINEER-FOR-A-DAY LABORATORY MODULES**

**ELECTRICAL ENGINEERING**  
**INTRODUCTORY LEVEL**  
**INTRODUCTION TO EXPONENTIALS AND NUMBER SYSTEMS**

**1.1 Exponentials**

Exponential notation is useful in situations where the same number is multiplied repeatedly.

The number being multiplied is called the base, and the exponent is the number of times the base is multiplied by itself.



**Example 1:** Convert the following multiplication into exponential form.

$$2 \times 2 \times 2 = \underline{\hspace{2cm}}$$

**Solution:**

1. The number being multiplied 2 is called the base.
2. The exponent 3 is the number of times the base is multiplied by itself.
3. Exponential form of base 2 and exponential 3 is  $2^3$

**Example 2:** Find the base and the exponent of the following exponential. Rewrite the exponential in expanded form and solve for it.

$$4^2 = \underline{\hspace{2cm}}$$

Base =    

Exponent =    

**Solution:**

1. Base = 4.
2. Exponent = 2.
3. To expand an exponential:  
Multiply the *base* by itself *exponent number of* times.
4. Therefore, multiply 4 (*the base*) by itself 2 (*exponent number of*) times
5. Expanded form =  $4 \times 4 = 16$ .

**Question 1:** Convert the following multiplication into exponential form.

$$6 \times 6 \times 6 \times 6 = \underline{\hspace{2cm}}.$$

**Solution:**

1. The number being multiplied 6 is called the base.
2. The exponent 4 is the number of times the base is multiplied by itself.
3. Exponential form of base 6 and exponential 4 is **6<sup>4</sup>**

**Question 2:** Find the base and the exponent of the following exponential.  
Rewrite the exponential in expanded form and solve for it.

$$5^6 = \underline{\hspace{2cm}}$$

Base =     

Exponent =     

**Solution:**

1. Base = 5
2. Exponent = 6
3. To expand an exponential:  
Multiply the *base* by itself *exponent number of* times.
4. Therefore, multiply 5 (*the base*) by itself 6 (*exponent number of*) times
5. Expanded form = **5 x 5 x 5 x 5 x 5 x 5 = 15625.**

**Question 3:** Find the base and the exponent of the following exponential.  
Rewrite the exponential in expanded form and solve for it.

$$12^3 = \underline{\hspace{2cm}}.$$

Base =     

Exponent =     

**Solution:**

1. Base = 12
2. Exponent = 3
3. To expand an exponential:  
Multiply the *base* by itself *exponent number of* times.
4. Therefore, multiply 12 (*the base*) by itself 3 (*exponent number of*) times
5. Expanded form = **12 x 12 x 12 = 1728.**

## 1.2 Decimal Numbers Places

Whole decimal numbers have a value, which depends on the place of each digit in the number. The decimal number places are ones, tens, hundreds, and so on. The following examples show the decimal place value for various positions.

**Example 3:** Place each digit of the decimal number given below in the right place of the table.

**4589**

Thousands	Hundreds	Tens	Units

**Solution:**

1. Inspect the number from right to left.
2. The right most number is in the units place or box.
3. The next number to the left is in tens place or box.
4. The number left to tens is in hundreds place or box.
5. The leftmost number is in thousands place or box.

Thousands	Hundreds	Tens	Units
4	5	8	9

**Question4:** Place each digit of the decimal number given below in the right place of the table.

**9532.**

Thousands	Hundreds	Tens	Units

**Solution:**

1. We start looking at the number from right to left.
2. The right most number is in units place.
3. Going left, the next number is in tens place.
4. The number left to tens is in hundreds place.
5. The leftmost number is in thousands place.

Thousands	Hundreds	Tens	Units
9	5	3	2

### 1.3 Place Value

Another way of writing numbers to show their place value is by expanded form. Every place in the decimal system, and in any other number system, has an exponential related to it. The exponential is written as follows:

1. The base, which is determined by the number system that is being used, in this case 10.
2. The exponential, which is determined by the place of each digit in the whole number for example units or tens, for the decimal number system.

Here is the relationship between the place value and the assigned exponential

Thousands	Hundreds	Tens	Units
1000	100	10	1
$10^3$	$10^2$	$10^1$	$10^0$

**Example 4:** Show the following decimal number in expanded form:

**4589** = \_\_\_\_\_.

**Solution:**

1. Since the number given above is in decimal number system, all the digits of the number are multiplied by base 10 to the respected place power.  
For instance:
  - The number at the units place gets multiplied by  $10^0$ , and the number at the tens place will be multiplied by  $10^1$  and so on.
2. After multiplying all the digits with the exponential, add all the products to get the final number. This is shown below:

Thousands		Hundreds		Tens		Units
4		5		8		9
4000	+	500	+	80	+	9
$4 \times 1000$	+	$5 \times 100$	+	$8 \times 10$	+	$9 \times 1$
$4 \times 10^3$	+	$5 \times 10^2$	+	$8 \times 10^1$	+	$9 \times 10^0$

$$4589 = (4 \times 10^3) + (5 \times 10^2) + (8 \times 10^1) + (9 \times 10^0).$$

**Question5:** Show the following decimal number in expanded form:

$$593 = \underline{\hspace{2cm}}$$

**Solution:**

1. Since the number given above is in decimal number system, all the digits of the number are multiplied by base 10 to the respected place power.  
For instance:
  - The number at the units place get multiplied by  $10^0$ , and the number at the tens place will be multiplied by  $10^1$  and so on.
2. After multiplying all the digits with the exponential, add all the products to get the final number. This is shown below:

Thousands		Hundreds		Tens		Units	
0		5		9		3	
0	+	500	+	90	+	3	
$0 \times 1000$	+	$5 \times 100$	+	$9 \times 10$	+	$3 \times 1$	
$0 \times 10^3$	+	$5 \times 10^2$	+	$9 \times 10^1$	+	$3 \times 10^0$	

$$593 = \underline{(5 \times 10^2) + (9 \times 10^1) + (3 \times 10^0)}.$$

**Question6:** Show the following decimal number in expanded form:

$$36431 = \underline{\hspace{2cm}}$$

**Solution:**

1. Since the number given above is in decimal number system, all the digits of the number are multiplied by base 10 to the respected place power.  
For instance:
  - The number at the units place gets multiplied by  $10^0$ , and the number at the tens place will be multiplied by  $10^1$  and so on.
2. After multiplying all the digits with the exponential, add all the products to get the final number. This is shown below:

$$36431 = \underline{(3 \times 10^4) + (6 \times 10^3) + (4 \times 10^2) + (3 \times 10^1) + (1 \times 10^0)}.$$

## 2.1 Number Systems

We refer to Number Systems as the set of number we use to count or keep records in a specific situation. For example the most common number system is the decimal number system we use every day, from 0 to 9. In our regular vocabulary we also referred to a based twelve number system for example, when ordering a dozen donuts. Computers are also a great example of number systems, since they base all their computation on a binary number system, which means only two numbers.

Let's think of some other numbers system, as the example indicates.

**Example 5:** List all the possible digits in the following systems:

**Base 4 Number System**=\_\_\_\_\_.

**Base 2 Number System**=\_\_\_\_\_ (also called binary system)

**Solution:**

1. The number of digits present in a number system is as many as the base of the system.
2. In this case there are 4 digits in base 4 and 2 digits in base 2.
3. The digits start from 0 and goes upto the base number minus one, for instance it would go from zero to three for base four and zero to one for base two.
4. If we run out of numbers we use letters.
5. So in Base 4 Number System, the digits included are 0, 1, 2, and 3.
6. In Base 2 Number System the digits included are 0 and 1.

**Question 7:** List all the possible digits in the following systems:

**9**= \_\_\_\_\_.

**10**=\_\_\_\_\_.

**5** = \_\_\_\_\_.

**13**=\_\_\_\_\_.

**Solution:**

1. The number of digits present in a number system is as many as the base of the system.
2. The digits start from 0 and goes upto the base number minus one for instance it would go from zero to twelve for base thirteen. So the digits included in the following Base Systems are:

$$9 = \underline{0,1,2,3,4,5,6,7,8}$$

$$10 = \underline{0,1,2,3,4,5,6,7,8,9}$$

$$5 = \underline{0,1,2,3,4}$$

$$13 = \underline{0,1,2,3,4,5,6,7,8,9,A,B,C}$$

**2.2 Number Systems Place Values**

As we already discussed for decimal number, each digit of a number regardless of its base holds a value. The procedures to determine this value are the same as in the decimal system:

1. Find the base. This is determined by the number system that is being used, for instance decimal would use base 10 and octal base 8.
2. Add the exponential. This is determined by the place of each digit in the whole number for example units or tens, for the decimal number system.

**Example 6:** Expand the number in various number systems

$$4573(\text{base } 8) = \underline{\hspace{10em}}.$$

$$4132(\text{base } 5) = \underline{\hspace{10em}}.$$

**Solution:**

1. Determine the base used for each number according to its Number System. In this case 8 and 5 respectively.
2. Determine the exponent corresponding to each place of the specified Number System. In this case  $8^0$  (units),  $8^1$  (eights),  $8^2$  (sixtyfours) and so on for base 8 numbers. For base 5 numbers it will be:  $5^0$  (units),  $5^1$  (fives),  $5^2$  (twentyfives) and so on.
3. All the products will be added to get the final number.
4. To expand the above number in base 8 multiply each digit by the exponential as follows:  
$$4573 = \underline{(4 \times 8^3) + (5 \times 8^2) + (7 \times 8^1) + (3 \times 8^0)}.$$
5. To expand the above number in base 5 multiply each digit by the exponential as follows:

$$4132 = \underline{(4 \times 5^3) + (1 \times 5^2) + (3 \times 5^1) + (2 \times 5^0)}.$$

**Question8:** Expand the number in various number systems

$$453(\text{base } 9) = \underline{\hspace{2cm}}$$

$$4132(\text{base } 6) = \underline{\hspace{2cm}}$$

**Solution:**

1. Determine the base used for each number according to its Number System. In this case it would be 9 and 6 respectively.
2. Determine the exponent corresponding to each place of the specified Number System. In this case  $9^0$  (units),  $9^1$  (nines),  $9^2$  (eightyones) and so on for base 9 numbers. For base 6 numbers it will be:  $6^0$  (units),  $6^1$  (sixs),  $6^2$  (thirtysixs) and so on.
3. All the products will be added to get the final number.
4. To expand the above number in base 9 multiply each digit by the exponential as follows:  
$$453 = (4 \times 6^2) + (5 \times 6^1) + (3 \times 6^0).$$
5. To expand the above number in base 6 multiply each digit by the exponential as follows:  
$$4132 = (4 \times 6^3) + (1 \times 6^2) + (3 \times 6^1) + (2 \times 6^0).$$