

**ENGINEERING GATORTRAX MATH EXCELLENCE PROJECT
ENGINEER-FOR-A-DAY LABORATORY MODULES**

**ELECTRICAL ENGINEERING
INTERMEDIATE LEVEL
BOOLEAN ALGEBRA**

1.0 Introduction to Boolean Algebra

Boolean algebra is all about logic. It is a system which uses algebraic notation to express logical relationships, just as conventional algebra is used to express mathematical relationships. It is named after George Boole (born 1815) who first formalized the mathematics of logic in the 1850's. In Boolean algebra the variables of an expression do not represent numbers but statements, and the logical operators which relate to them such as "Or", "And", and "Not". In a simplification of Boolean algebra, the values of the variables are limited to two values (known as the *Truth Values*) "True" and "False".

In Boolean algebra the most common symbol for "true" is 1; for "false", 0.

1.1 Propositions

A proposition is something which is either true or false.

Examples of Propositions

1. It is cold in Alaska.
2. Washington is the Capitol of United States.
3. $5 + 8 = 10$.

Examples of Non Propositions

1. What is your name?
2. Oh my God!
3. $x + 5 = 3$

2.1 Combining Propositions

The words AND, OR, and NOT are part of our daily vocabulary. In fact they are a very important element of our decision-making strategies.

In Boolean Algebra these words are also determine on the final value of and operation, called the truth value, when combining two or more propositions.

Since the words AND, OR and NOT are part of our daily vocabulary, we use them to represent important elements of our decision-making strategies. As indicated above, these words also determine the final value of an operation, otherwise called the Truth Value, when combining two or more propositions.

Example:

p: $4 < 5$

q: $8 + 5 = 10$

p AND q are FALSE

p OR q is TRUE

NOT q is TRUE

2.2 Boolean Operators

Considering the three Boolean operators:

- NOT - the negation operator returns opposites of operands
- AND - yields true values, if and only if both operands are true
- OR - yields true values if either (or both) operands are true

The *Truth Tables* demonstrate use of these operators.

Truth Tables

Table 1. Use of the "NOT" Operator

<u>a</u>	<u>NOT (a)</u>
TRUE	FALSE
FALSE	TRUE

Table 2. Use of "AND" and "OR" Operators

<u>a</u>	<u>b</u>	<u>a AND b</u>	<u>a OR b</u>
TRUE	TRUE	TRUE	TRUE
TRUE	FALSE	FALSE	TRUE
FALSE	TRUE	FALSE	TRUE
FALSE	FALSE	FALSE	FALSE

STORY

One cold night Maria heard her baby crying. She got up and went to the baby's room. The baby was crying because he was scared. Looking at the mother the baby stooped crying and went to sleep. The mother slept with the baby in the baby's room. Next morning the mother was talking to the father about the baby using the following propositions:

P= The baby was crying.

Q= The baby never went to sleep.

Example1: Negate the following proposition, i.e. NOT P, and complete the truth table.

P= The baby was crying.
NOT P= _____

P	NOT P

Solution:

1. We simply have to take the proposition and (invert it) say opposite of it.
2. This will be done by adding the word not in the proposition
3. In this case the negation will be: "The baby was not crying."
Not P= The baby was not crying.
4. Since the original proposition **P** was truth these are the values of the truth table

P	NOT P
T	F

Example2: OR the following propositions, i.e.: P or Q, and complete the truth table.

P= The baby was crying.
Q= The baby never went to sleep.

P	Q	P OR Q

P OR Q = _____

Solution:

1. Two propositions are ORed by combining them with an "OR" in between the two of them.
2. We will simply say P or Q.
3. In other words "The baby was crying or the baby never went to sleep."
4. This can also be written as "The baby was crying or eventually went to sleep."
P OR Q= "The baby was crying or never went to sleep."
5. Since the one prepositions is truth and the other false these are the values of the truth table

P	Q	P OR Q
T	F	T

Example3: **AND** the following propositions, i.e.: P and Q, and complete the truth table.

P= The baby was crying.
Q= The baby never went to sleep.
P AND Q = _____

P	Q	P AND Q

Solution:

- Two propositions are AND by combining them with an "AND" in between the two of them.
- We will simply say P and Q.
- In other words "The baby was crying and the baby never went to sleep."
- This can also be written as "The baby was crying and never went to sleep."
P AND Q= The baby was crying and never went to sleep.
- Since the one propositions is truth and the other false this are the values of the truth table

P	Q	P AND Q
T	F	F

Exercises

A= January is the first month of the year.
B= It is hot in January.
L= Postman delivers the mail
M= 5 +2=3

Question 1: **Negate** the following proposition, and complete the truth table.

A= January is the first month of the year.
NOT A= January is not the first month of the year

A	NOT A

Solution:

- We will add the word not in the proposition to negate it
- "January is not the first month of the year."
- Since the original proposition **A** was truth this are the values of the truth table

A	NOT A
T	F

Question 2: **Negate** the following proposition:

B= It is hot in January.
NOT B= It is not hot in January.

B	NOT B

Solution:

4. We will add the word not in the proposition to negate it
5. "It is not hot in January."
6. Since the original proposition **B** was false this are the values of the truth table

B	NOT B
F	T

Question 3: OR the following propositions:

A= January is the first month of the year.
B= It is hot in January.

A	B	A OR B

A OR B= January is the first month of the year or it is hot in January

Solution:

1. We will say A or B.
2. January is the first month of the year or it is hot in January.
3. Since the one propositions is truth and the other false these are the values of the truth table

A	B	A OR B
T	F	T

Question 4: OR the following propositions:

L= Postman delivers the mail
M= $5 + 2 = 3$

L	M	L OR M

L OR M= Postman delivers the mail or $5 + 2 = 3$.

Solution:

4. We will say A or B.
5. Postman delivers the mail or $5 + 2 = 3$.
6. Since the one propositions is truth and the other false these are the values of the truth table

L	M	L OR M
T	F	T

Question 5: AND the following propositions:

L= Postman delivers the mail

M= 5 +2=3

L AND M= Postman delivers the mail and 5+2=3.

L	M	L AND M

Solution:

1. We need to say L and M.
2. Postman delivers the mail and 5+2=3.
3. Since the one prepositions is truth and the other false these are the values of the truth table

L	M	L AND M
T	F	T

Question 6: AND the following propositions:

A= January is the first month of the year.

B= It is hot in January.

A AND B= January is the first month of the year and it is hot in January

A	B	A AND B

Solution:

1. We need to say A and B.
2. January is the first month of the year and it is hot in January.
4. We can simplify and say "January is the first month of the year and it is hot."
5. Since the one prepositions is truth and the other false these are the values of the truth table

A	B	A AND B
T	F	F

The Problem:

One very sunny Friday morning, 8 students from Goonesville High decided to skip school and head to the beach for a day of sunbathing, surfing and water- skiing. Unfortunately for them the school 's resource officer, Officer Doom decided to do a beach patrol on that day. He spotted the students, carefully noted who they were and reported them to the principal.

The following Monday the students were summoned to the principal's office to account for their absence from school on Friday. Some of them had time to meet before seeing the principal in order to establish what they would say. However, they could not find a couple of those who had skipped with them, and these were unaware of the story which had been agreed should be told to the principal.