**ENGINEERING GATORTRAX MATH EXCELLENCE PROJECT**
**ENGINEER-FOR-A-DAY LABORATORY MODULES**

**ELECTRICAL ENGINEERING**
**ADVANCE LEVELS**
**INTRODUCTION TO LINEAR EQUATIONS**

## 1.0 Algebraic Equations

An **equation** is a statement that two algebraic expressions are equal. For example, $3x-5=7$, $x^2-x-6=0$, and $\sqrt{2x}=4$, are equations. To **solve** an equation in $x$ means to find all values of x for which the equation is true. Such values are **solutions**. For instance, $x=4$ is a solution of the equation $3x-5=7$, because $3(4)-5=7$ is a true statement.

### 1.1 Linear Equations in One Variable

A **linear equation** in one variable $x$ is an equations that can be written in the standard form
$ax+b=0$
where $a$ and $b$ are real numbers with $a\neq 0$.

**Generating Equivalent Equations**
An equation can be transformed into an equivalent equation by one or more of the following steps:

| | Given Equation | Equivalent Equation |
|---|---|---|
| 1. Remove symbols of grouping, combine like terms, or reduce fractions on one or both sides of the equations. | $2x-x=4$ | $x=4$ |
| 2. Add (or subtract) the same quantity to both sides of the equation. | $x+1=6$ | $x=5$ |
| 3. Multiply (or divide) *both* sides of the equation by the same *nonzero* quantity. | $2x=6$ | $x=3$ |
| 4. Interchange the two sides of the equation. | $2=x$ | $x=2$ |

**<u>Example 1:</u> Solving a Linear Equation**

| | |
|---|---|
| $3x-6=0$ | Given equation |
| $3x=6$ | Add 6 to both sides |
| $x=2$ | Divide both sides by 3 |

 

After solving an equation, you should **check each solution** in the *original* equation. In Example 1, check that 2 is a solution by substituting 2 for $x$ in the original equation.
3(2)-6= 6-6=0 Check solution
Some linear equations have no solutions because all the $x$-terms subtract out and a contradictory (false) statement such as $0=5x$ or 12=7 is obtained. Watch for this type of linear equation in the exercises.

**Example 2:** **Solving a Linear Equation**

| | |
|---|---|
| $6(x-1)+4=3(7x+1)$ | Given equation |
| $6x-6+4=21x+3$ | Remove parenthesis |
| $6x-2=21x+3$ | Simplify |
| $-15x=5$ | Add 2 and subtract $21x$ |
| $x=\frac{-1}{3}$ | Divide by -15 |

Now we are going to solve for a technical linear equation. This is equation is call Ohm's Law and is one of the most important equation in electrical engineering

**OHM'S LAW:**

VOLTAGE= RESISTANCE * CURRENT
Or

$$\mathbf{V} = \mathbf{R} \times \boldsymbol{i}$$

We could also express this equation in $ax+b=0$ format by performing a few operation on it:

$$ax + b = 0$$

$$\mathbf{R}\boldsymbol{i} + (-\mathbf{V}) = \mathbf{0}$$

**Exercise 1:**

GIVEN : $i_1$= 5 mA **$R_1$**= 1 kΩ **$V_0$** = $R_1 \times i_1$
SOLVE FOR: **$V_0$**

METRIC PREFIXES

| | | |
|---|---|---|
| kilo | k | $10^3$ |
| hecto | h | $10^2$ |
| deka | da | $10^1$ |
| UNIT | 1 | $10^0$ |
| deci | d | $10^{-1}$ |
| centi | c | $10^{-2}$ |
| milli | m | $10^{-3}$ |

**Solution:**

$$V_0 = R_1 \times i_1$$
$$V_0 = 1\ k\Omega \times 5\ mA$$
$$= 5V$$

## 1.2 Equation Involving Fractional Expressions

To solve an equation involving fractional expressions, we find the least common denominator of all terms in the equation and multiply every term by this LCD. This procedure clears the equation of fractions.

**Example 3: Solving an Equation Involving Fractional Expressions**
Solve the equation for *x*.

$$\frac{x}{3} + \frac{3x}{4} = 2$$

**Solution**

| | |
|---|---|
| $\frac{x}{3} + \frac{3x}{4} = 2$ | Given equation |
| $(12)\frac{x}{3} + (12)\frac{3x}{4} = (12)2$ | Multiply by the LCD |
| $4x + 9x = 24$ | Reduce and multiply |
| $13x = 24$ | Combine like terms |
| $x = \frac{24}{13}$ | Divide by 13 |

The equation has one solution: $\frac{24}{13}$. Check this solution in the original equation.

**Exercise 2:**

GIVEN : $R_2 = 1\ k\Omega$ $R_3 = 1\ k\Omega$ $V_o = 5V$

$i_1 = i_2$

$i_1 = \frac{V_0 - V_1}{R_2}$ $i_2 = \frac{V_1}{R_3}$

SOLVE FOR: $V_1$

 

**Solution:**

$$i_1 = i_2$$

$$\frac{V_0 - V_1}{R_2} = \frac{V_1}{R_3}$$

$$\frac{V_0 - V_1}{\mathbf{1k}} = \frac{V_1}{\mathbf{1k}}$$

$$V_o - V_1 = V_1$$

$$V_1 = \frac{V_0}{2} = \frac{5\ V}{2} = 2.5\ V$$

| ANSWER TABLE 2 | |
|---|---|
| $\mathbf{V_0}$ | 5 V |
| $\mathbf{V_1}$ | 2.5 V |

**Exercise 3:**

GIVEN : $\mathbf{R_2 =}$ 1 kΩ $\mathbf{R_3 =}$ 1 kΩ $\mathbf{R_4 =}$ 2.2 kΩ **Vo =** 5V

$$i_1 + i_3 = i_2$$

$$i_1 = \frac{V_0 - V_1}{R_2} \qquad i_2 = \frac{V_1}{R_3} \qquad i_3 = \frac{V_0 - V_1}{R_4}$$

SOLVE FOR: $\mathbf{V_1}$

| ANSWER TABLE 3 | |
|---|---|
| $\mathbf{V_0}$ | 5 V |
| $\mathbf{V_1}$ | 2.96 V |

 

**Solution:**

$$i_1 + i_3 = i_2$$

$$\frac{V_0 - V_1}{R_2} + \frac{V_0 - V_1}{R_4} = \frac{V_1}{R_3}$$

$$\frac{V_0 - V_1}{1k} + \frac{V_0 - V_1}{2.2k} = \frac{V_1}{1k}$$

$$\frac{2.2(V_0 - V_1) + V_0 - V_1}{2.2k} = \frac{V_1}{1k}$$

$$2.2(V_0 - V_1) + V_0 - V_1 = 2.2\,V_1$$

$$2.2V_0 - 2.2V_1 + V_0 - V_1 = 2.2V_1$$

$$-3.2V_1 + 3.2V_0 = 2.2V_1$$

$$3.2V_0 = 5.4\,V_1$$

$$V_1 = \frac{3.2 \times 5V}{5.4}$$

$$V_1 = 2.963\text{ V}$$